Complex Analysis Prelim Exam

UC Department of Math
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1. Show that every entire function is given by a power series that converges locally uniformly on the complex plane. By an entire function we mean a function that is complex analytic on the entire plane.

2. For a given integer $j \in \mathbb{Z}$, find all entire functions f that satisfy $|f(z)| \leq |e^z|\,|z-i|^j$ for each complex number $z \neq i+1$.

3. In this question, Ω is a simply connected planar domain that is not the entire complex plane, and $z_0, z_1 \in \Omega$. Show that if f and g are two conformal maps of Ω that map z_0 to z_1, then $f = g$.

4. Compute the exact value of the integral $\displaystyle\int_0^\infty \frac{1}{x^{4n}+1}\,dx$ for each positive integer n.

5. Let D be the open disk $D = \{z : |z-c| < \rho\}$, where $c, \rho \in \mathbb{R}$ with $0 < \rho < c$ and let H denote the left half-plane $H = \{z : Re(z) < 0\}$. Find the image of the union $D \cup H$ under the mapping
$$z \to \frac{z-a}{z+a}$$
where $a = \sqrt{c^2 - \rho^2} > 0$.