1. Let $(X_n)_{n=1}^{\infty}$ be a sequence of random variables which converges to 0 in probability as $n \to \infty$.

   (a) Let $M > 0$ be a fixed number. Show that
   $$\lim_{n\to\infty} E[|X_n|\mathbf{1}_{|X_n|\leq M}] = 0.$$

   (b) Show that, if there exists a constant $C > 0$ such that $E[|X_n|^{\alpha}] \leq C$ for all $n \geq 1$, then we have $\lim_{n\to\infty} E[|X_n|] = 0$.

2. Let $(Y_k)_{k=1}^{\infty}$ be a sequence of independent random variables with
   $$P(Y_k = 1 + \sqrt{k}) = P(Y_k = 1 - \sqrt{k}) = \frac{1}{2}, k \in \mathbb{N}.$$

   (a) Compute $E[Y_k]$ and $\mathrm{Var}(Y_k)$.

   (b) Show that there exists two sequences of constants (a_n), (b_n) such that, as $n \to \infty$,
   $$\frac{Y_1 + \ldots + Y_n - a_n}{b_n}$$
   converges in distribution to a standard normal random variable.

3. Let $(X_k)_{k=1}^{\infty}$ be i.i.d. random variables with probability density function
   $$f(x) = \begin{cases} 2x^{-3} & \text{when } x \geq 1 \\ 0 & \text{when } x < 1. \end{cases}$$

   (a) Prove that $X_n/\sqrt{n}$ converges to 0 in probability as $n \to \infty$.

   (b) Prove that $X_n/\sqrt{n}$ does not converge to 0 almost surely as $n \to \infty$.

4. Let $(X_k)_{k=1}^{\infty}$ be the same i.i.d. sequence of random variables as in Problem 3. Let $M_n = \max_{k=1,\ldots,n} X_k$ denote the maximum of $X_1, \ldots, X_n$.

   Prove that $M_n/\sqrt{n}$ converges in distribution to a random variable Z, and identify the distribution of Z by finding its probability density function.