1. Suppose X and $(X_n)_{n\in\mathbb{N}}$ are non-negative random variables such that X_n converges to X in probability as $n \to \infty$. Does it always hold that $\lim_{n\to\infty} \mathbb{E}X_n^2 = \mathbb{E}X^2$?

   If your answer is yes, provide a proof. If your answer is no, provide a counter example.

2. Let $(X_n)_{n=1}^{\infty}$ be i.i.d. random variables with probability density function

$$f(x) = \begin{cases} 3(1+x)^{-4}, & \text{if } x > 0, \\ 0, & \text{if } x \leq 0. \end{cases}$$

   Let $\beta > 0$ be a parameter.

   (a) Find all β so that X_n/n^{β} converges to zero in probability as $n \to \infty$.

   (b) Find all β so that X_n/n^{β} does not converge to 0 almost surely as $n \to \infty$.

   Justify your answers.

3. Let $(X_n)_{n=1}^{\infty}$ be a sequence of random variables such that

$$\mathbb{P}(X_n = k) = \frac{1}{c_n}(n-k), k = 0, 1, \ldots, n-1,$$

   where $c_n = \sum_{k=0}^{n-1}(n-k)$. Show that X_n/n converges in distribution as $n \to \infty$, and identify the cumulative distribution function of the limit.

4. Let $(X_i)_{i\in\mathbb{N}}$ be i.i.d. random variables with $\mathbb{E}X_1 = 0, \mathbb{E}X_1^2 = 1$. Set

$$a_i := \begin{cases} 1, & \text{if } i \text{ is odd}, \\ 2, & \text{otherwise}. \end{cases}$$

   Set $S_n := \sum_{i=1}^{2n} a_i X_i$. Find a sequence of $(c_n)_{n\in\mathbb{N}}$ such that S_n/c_n converges in distribution to a standard Gaussian random variable. Justify your answer.