Algebra and Topology Preliminary Exam Syllabus
Department of Mathematical Sciences
University of Cincinnati

Algebra

Rings, homomorphisms and ideals, quotient rings, integral domains and fraction fields, prime and irreducible elements. Unique factorization domains, principal ideal domains and Euclidean domains, Gauss' lemma. Fields and field extensions, algebraic and transcendental elements, adjunction of roots, finite fields. Galois theory: splitting fields, normal and separable extensions, the Main Theorem of Galois theory. Cyclic and cyclotomic extensions, solvable and radical extensions, insolvability of the quintic equation.

This material is covered in MATH7003

Texts: Dummit and Foote, Abstract Algebra, Parts II and IV; Lang, Algebra; Stewart, Galois Theory. Artin, Galois Theory.

Topology

Point-set topology: Topological spaces, closed sets, subspaces, closure, boundary, interior, connectedness, pathconnectedness, compactness, normal topology, Hausdorff property, continuity at a point (topological continuity and sequential continuity), continuous maps, Urysohn metrization theorem, Tietze extension theorem, quotient topology, weak topology, Baire category theorem, nets, convergence with respect to nets. Fundamental groups: Homotopy of paths, homotopy of maps, fundamental groups, fundamental groups of (i) circles, (ii) spheres, (iii) tori, (iv) Möbius strip and (v) Klein bottle, free groups, simply connected spaces, covering spaces, homotopy lifting theorem.

This material is covered in MATH7004

Texts:
James R. Munkres, Topology (second edition), Prentice Hall, 2000.
Allen Hatcher, Algebraic Topology, Cambridge University Press, 2002 (Chapters 0-1);
William S. Massey, A basic Course in Algebraic Topology, Springer, 1991 (Chapters 1-5).