Name:____________________ M#:__________ Instructor:____________________

Instructions: This exam has seven problems on seven pages. Show all your work, expressing yourself in clear and concise manner. Do as many problems as you can, but be advised that a complete solution to a problem may be worth more than several partial ones. Use the backs of the exam pages for work, if necessary. No calculators of any kind are allowed.

1. Compute the limit or show that it does not exist.

$$\lim_{x\to\infty} e^{\sin(2x)-x}$$

2. Let $A = (-3, 7)$, $B = (1, 3)$, and $C = (5, 3)$ be three points in the plane. Sketch the triangle ABC and find the volume of the solid obtained by revolving the triangle around the line containing the altitude from the point A.

3. Does the series $\sum_{n=1}^{\infty} n\, e^{-\sqrt{n}}$ converge or diverge? Fully justify your answer.

4. Compute

$$\lim_{m\to\infty} \sum_{n=1}^{m} \frac{n^5}{m^6}$$

(Hint: Riemann sums.)

5. Let

$$f(x) = \begin{cases} xe^{-\frac{1}{x^2}}, & x \neq 0, \\ a, & x = 0. \end{cases}$$

(a) Find a so that f is continuous for all x.

(b) Using your answer for part (a), show that f is differentiable at $x = 0$ and find $f'(0)$.

6. Compute

$$\int_0^\infty \frac{1}{1+x^3}\,dx$$

(Hint: use partial fractions; one factor of x^3+1 is $x+1$.)

7. A function f on the real line is called mid-point convex if for any real numbers x_1 and x_2 one has
$$f\left(\frac{x_1+x_2}{2}\right) \leq \frac{1}{2}\, f(x_1) + \frac{1}{2}\, f(x_2).$$
Show that if f is twice continuously differentiable and $f''(x) \geq 0$ for all x, then f is mid-point convex.

   (Hint: what happens if you approximate $f(x_1)$ and $f(x_2)$ by the corresponding first degree Taylor polynomials centered at the point $x_0 = \frac{x_1+x_2}{2}$?)