- 1. Let **L** be a non-singular linear transformation. Prove that if $\mathbf{A} = \mathbf{L}\mathbf{M}\mathbf{L}^{-1}$, then $e^{\mathbf{A}t} = \mathbf{L}e^{\mathbf{M}t}\mathbf{L}^{-1}$.
- 2. Consider the equation

$$\dot{x} = x^4 + rx^2 + x^2 + 1.$$

- (a) Sketch the phase line, identify the equilibrium, and determine stability for representative values of r.
- (b) Draw the bifurcation diagram and identify the type of bifurcation as r varies.
- 3. Show that the rest point (1,1) is asymptotically stable for the system

$$x' = -3x - y + xy + 3$$

$$y' = -2x - y + x^{2} + 2$$

4. Consider the system

$$x' = 1 - y^2$$
$$y' = 1 - x^2$$

- (a) Determine all equilibria and classify.
- (b) Sketch the phase portrait in detail.
- 5. Assume that the functions $a(x) \ge 0$ and $u(x) \ge 0$ are continuous for $x \ge x_0$ and that we have

$$u(x) \le \int_{x_0}^x a(t) u(t) dt$$

for $x \ge x_0$. Show that u(x) = 0 for $x \ge x_0$.