1. Let $\mathbf{L}$ be a non-singular linear transformation. Prove that if $\mathbf{A}=\mathbf{L} \mathbf{M L} \mathbf{L}^{-1}$, then $e^{\mathbf{A} t}=\mathbf{L} e^{\mathbf{M} t} \mathbf{L}^{-1}$.
2. Consider the equation

$$
\dot{x}=x^{4}+r x^{2}+x^{2}+1
$$

(a) Sketch the phase line, identify the equilibrium, and determine stability for representative values of $r$.
(b) Draw the bifurcation diagram and identify the type of bifurcation as $r$ varies.
3. Show that the rest point $(1,1)$ is asymptotically stable for the system

$$
\begin{aligned}
x^{\prime} & =-3 x-y+x y+3 \\
y^{\prime} & =-2 x-y+x^{2}+2
\end{aligned}
$$

4. Consider the system

$$
\begin{aligned}
& x^{\prime}=1-y^{2} \\
& y^{\prime}=1-x^{2}
\end{aligned}
$$

(a) Determine all equilibria and classify.
(b) Sketch the phase portrait in detail.
5. Assume that the functions $a(x) \geq 0$ and $u(x) \geq 0$ are continuous for $x \geq x_{0}$ and that we have

$$
u(x) \leq \int_{x_{0}}^{x} a(t) u(t) d t
$$

for $x \geq x_{0}$. Show that $u(x)=0$ for $x \geq x_{0}$.

