# Preliminary Exam Problem Suggestions

Complex Analysis. Fall 2021

1. Prove that a non-constant entire function $f$ cannot satisfy both $f(z+1) = f(z)$ and $f(z+\mathrm{i}) = f(z)$ for all $z \in \mathbb{C}$.

2. Suppose that $f\colon \mathbb{D} \to \mathbb{C}$ is holomorphic and that $f$ is injective in some annulus $A = \{z\colon r < |z| < 1\}$, with $r > 0$. Show that $f$ is injective in $\mathbb{D}$.

3. Find a Laurent series expansion of $f(z) = \dfrac{1}{(z-1)(z-2)}$ that is valid in the annulus $1 < |z| < 2$.

4. Use the residue theorem to compute

$$\int_{-\infty}^{+\infty} \frac{1}{(x^2+1)^2}\,\mathrm{d}x.$$

5. Find a conformal map that maps the region $\{z \in \mathbb{C} : Re(z) > 0, 0 < Im(z) < 4\pi\}$ to the half-disk $\{z \in \mathbb{C} : |z| < 1, Im(z) > 0\}$ and maps $1+i$ to $i/2$. (Hint: Think about exponential functions)

6. Suppose $g : \mathbb{C} \to \mathbb{C}$ is a holomorphic function, $k$, $n$ are integers and

$$|g^{(n)}(z)| \leq 3|z|^k \text{ when } |z| \geq 1,$$

show $g$ is a polynomial and estimate the degree of $g$ in terms of $k$, $n$.