1. Prove the following property of the matrix exponential:

$$
\frac{d}{d t}\left[e^{\mathbf{A} t}\right]=\mathbf{A} e^{\mathbf{A} t}
$$

where $\mathbf{A} \in \mathbb{C}^{N \times N}$ and $t \in \mathbb{C}$.
2. Consider the equation

$$
\dot{x}=r x e^{-x^{2}}-x\left(1+x^{2}\right) .
$$

(a) Sketch the phase line, identify the equilibrium, and determine stability for representative values of $r$.
(b) Draw the bifurcation diagram and identify the type of bifurcation as $r$ varies.
3. Consider the system

$$
\begin{aligned}
\dot{x} & =x-y-(x+y)\left(x^{2}+y^{2}\right) \\
\dot{y} & =x+y+(x-y)\left(x^{2}+y^{2}\right)
\end{aligned}
$$

(a) Determine all the equilibria and their stability.
(b) Show there exists a stable periodic orbit and state where.
4. Consider the equation

$$
\ddot{x}=\left(1-x^{2}\right)\left(4-x^{2}\right) .
$$

(a) State the energy function $E(x, \dot{x})$.
(b) State and classify the equilibria.
(c) Sketch the phase portrait in detail.
5. Consider the system

$$
\begin{aligned}
& \dot{x}=a x+b y \\
& \dot{y}=c x+d y
\end{aligned}
$$

with constant $a, b, c$, and $d$. Assume that $a+d<0$ and $a d-b c>0$. Show that all solutions tend to zero as $t \rightarrow \infty$.

