Complex Analysis Prelim Exam
UC Department of Math
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1. Let
$$f(z) = y - 2xy + i(-x + x^2 - y^2) + z^2$$
   where $z = x + iy$. For what values of z does $f'(z)$ exists?

2. Use the Residue Theorem to compute
$$I = \oint_{|z|=2} \frac{z+2}{z(z+1)}\, dz,$$
   where the contour of integration is oriented counter-clockwise.

3. Consider the horizontal strip $S = \left\{ z \colon -\frac{\pi}{2} < \text{Im}(z) < \frac{\pi}{2} \right\}$. Find all the conformal maps that maps S to the open unit disk and map 0 to 0 with $f'(0) > 0$. For partial credit find one such mapping.

4. Show that $e^z - (5z^2 + 1) = 0$ has exactly two roots in the open unit disk $|z| < 1$.

5. Show that an entire function f on $\mathbb{C}$ satisfying $|f(z)| \leq \sqrt{1 + |z|}$ for all $z \in \mathbb{C}$ is constant.