Mathematics Qualifying Exam Syllabus

Department of Mathematical Sciences

University of Cincinnati

(Updated August 2022)

Advanced Calculus Topics

Number Fields: Basic axioms and operations for real numbers, least upper bound property (completeness), sequences.

Topology: Open, closed, compact, and connected sets in $\mathbb{R}^n$ and their properties, Bolzano-Weierstrass and Heine-Borel theorems, cluster points of an infinite set.

Metric Spaces: Bounded and unbounded sets, convergent sequences, Cauchy sequences, completeness.

Series: Tests for convergence and absolute convergence.

Functions: Limits at cluster points, continuity via limits, ϵ-δ, and pre-images of open or closed sets, upper and lower limits (limit inferior and limit superior), continuity and compactness (Extreme Value Theorem), continuity and connectedness (Intermediate Value Theorem).

Differentiation: Limits and derivatives of functions on the real line, properties of derivatives (product and quotient rules, chain rule, Rolles' theorem, Mean Value Theorem, l'Hôpital's rule), higher order derivatives.

Function Sequences: Uniform vs. pointwise convergence, Weierstrass M-test, sequence of step functions converging uniformly to continuous functions on a compact interval, uniform convergence of continuous functions, uniform convergence of integrable functions.

Power Series: Radius of convergence, Taylor's series and theorem, analytic functions vs. smooth functions.

Integration: Riemann integrals on intervals on the real line, upper and lower Riemann sums, integrability of step functions, Riemann integrability of uniform limits of Riemann integrable functions, integrability of continuous functions, properties of integrals.

This material is covered in MATH 6001 Advanced Calculus I.

Texts:
Apostol, *Mathematical Analysis*, 2nd edition, 1974
Bartle, *The Elements of Real Analysis*, 2nd edition, 1991
Dence and Dence, *Advanced Calculus: A Transition to Analysis*, 2010
Folland, *Advanced Calculus*, 2002
Rosenlicht, *Introduction to Analysis*, 1986
Rudin, *Principles of Mathematical Analysis*, 3rd edition, 1976

Linear Algebra Topics

Vector Spaces: Fields, subspaces, isomorphisms, linear independence, bases, dimension.

Linear Mappings: Kernel, image, range, composition, inverses, similarities, projections.

Matrices: Homogenous and nonhomogenous linear equations, Gaussian elimination, null space, column and row spaces, rank, Rank-Nullity theorem, trace, matrix representative of a linear map, similarity of matrices, positive-definite matrices, inverse of a matrix.

Orthogonality: Norms and inner products, Schwarz and triangle inequalities, orthogonality, orthonormal bases, Gram-Schmidt procedure.

Determinants: Definition, properties, signed volume, Cramer's rule, relationship to invertibility.

Symmetry: Symmetric, Hermitian, and unitary matrices.

Spectral Theory: Eigenvectors and eigenvalues, characteristic polynomial, diagonalization of symmetric and Hermitian matrices, generalized eigenvectors and Jordan canonical form.

This material is covered in MATH 6003 Abstract Linear Algebra.

Texts:
Artin, *Algebra*, 2nd edition, 2010
Lang, *Linear Algebra*, 3rd edition, 2010