Statistics Qualifying Exam

September 14, 2011

9:00 – 1:00

Show All Work

1. Let X have probability density function (pdf), $f(x) = e^{-x}$ $0 < x < \infty$; 0 *elsewhere.* Find $M(t)$, the moment-generating function of X.
2. Players A and B play a sequence of games. Player A throws a die first and wins on a "six". If he fails, B throws the die and wins on a "five" or "six". If he fails, A throws the die and wins on a "four", "five", or "six". This goes on until someone wins. Find the probability that player B wins.
3. Let X_1 and X_2 have the joint probability density function (pdf), $f(x_1, x_2) = x_1 + x_2$, for $0 < x_1 < 1$, $0 < x_2 < 1$; 0 elsewhere. Find the conditional mean and variance of X_2, given $X_1 = x_1$, $0 < x_1 < 1$.
4. Let X have probability density function (pdf), $f(x) = \frac{1}{x^2}$ $1 < x < \infty$, 0 elsewhere. Consider a random sample of size 81 from the distribution having this pdf. Compute the probability that more than 50 of the observations are less than 3.
5. Let X have a binomial distribution with $n = 300$ and probability, p, i.e., $b(300, p)$. The observed value of X is $x = 75$.
   (a) Find an approximate 90% confidence interval for p.
   (b) Test the null hypothesis, $H_0: p = \frac{1}{3}$ v.s. the alternative hypothesis, $H_1: p < \frac{1}{3}$ at the $\alpha = 0.05$ level of significance.
6. Let X have a binomial distribution with parameters n and p. We reject the null hypothesis, $H_0: p = 0.5$ in favor of the alternative hypothesis, $H_1: p > 0.5$ if $X \geq c$. Find n and c to give a power function $\gamma(0.5) = 0.10$ and $\gamma(2/3) = 0.95$, approximately.
7. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables each having probability density function (pdf),

   $$(x) = \frac{1}{\theta_2} e^{-\frac{x-\theta_1}{\theta_2}}, \qquad \theta_1 \leq x < \infty, -\infty < \theta_2 < \infty$$

   , 0 elsewhere. Find the maximum likelihood estimators of θ_1 and θ_2.

8. A state highway department studied the wear characteristics of five different paints are eight locations in the state. The standard, currently used paint (paint 1) and four experimental paints (paints 2, 3, 4, 5) were included in the study. The eight locations were randomly selected, thus reflecting variations in traffic densities throughout the state. At each location, a random ordering of the paints to the chosen road surface was employed. After a suitable period of exposure to weather and traffic, a combined measure of wear, considering both durability and visibility, was obtained. The data on wear follow.

location	Paint(j)				
i	1	2	3	4	5
1	11	13	10	18	15
2	20	28	15	30	18
3	8	10	8	16	12
4	30	35	27	41	28

location	Paint(j)				
i	1	2	3	4	5
5	14	16	13	22	16
6	25	27	26	33	25
7	43	46	41	55	42
8	13	14	12	20	13

Source	df	SS	MS	F
Blocks	7	4826.375		
Paint types	4	531.350		
Error		122.250		
Total	39			

(a) State an appropriate statistical model including model assumptions.

(b) Obtain the complete ANOVA table.

(c) Test whether or not the mean wear differs for the five paints; Use $\alpha = 0.05$. State the alternatives, decision rule, and conclusion.

(d) Paints 1, 3, and 5 are white, whereas paints 2 and 4 are yellow. Estimate the difference in the mean wear for the two groups of paints with a 95% confidence interval. Interpret your findings.

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$\bar{Y}_{\cdot 1} = 20.5, \bar{Y}_{\cdot 2} = 23.625, \bar{Y}_{\cdot 3} = 19.0, \bar{Y}_{\cdot 4} = 29.375, \bar{Y}_{\cdot 5} = 21.125$

9. Given the following information, with n=13.

Number in Model	R-Square	MSE	SSE	Variables in Model
1	0.6745	80.35154	883.86692	X4
1	0.6663	82.39421	906.33634	X2
1	0.5339	115.06243	1265.68675	X1
1	0.2859	176.30913	1939.40047	X3
2	0.9787	5.79045	57.90448	X1 X2
2	0.9725	7.47621	74.76211	X1 X4
2	0.9353	17.57380	175.73800	X3 X4
2	0.8470	41.54427	415.44273	X2 X3
2	0.6801	86.88801	868.88013	X2 X4
2	0.5482	122.70721	1227.07206	X1 X3
3	0.9823	5.33030	47.97273	X1 X2 X4
3	0.9823	5.34562	48.11061	X1 X2 X3
3	0.9813	5.64846	50.83612	X1 X3 X4
3	0.9728	8.20162	73.81455	X2 X3 X4
4	0.9824	5.98295	47.86364	X1 X2 X3 X4

(a) Test the hypothesis $H_0{:}\beta_1 = \beta_2 = 0$ v.s. $H_0{:}\beta_1 = 0$ at the level of significance $\alpha = 0.05$.

(b) Perform a forward stepwise regression using the level of significance $\alpha = 0.05$.

(c) Perform the backward elimination regression. You must justify each step using the level of significance $\alpha = 0.05$.