Statistics Qualifying Exam

1-5pm, Monday, September 15, 2008

Name :

1. Let the p.m.f. $p(x)$ be positive at $x = -1, 0, 1$ and zero elsewhere.

   (a) If $p(0) = \frac{2}{5}$, find $E(X^2)$.

   (b) If $p(0) = \frac{2}{5}$ and if $E(X) = \frac{1}{3}$, determine the p.m.f. of X.

2. Let $X_1, \ldots, X_4$ be *i.i.d.* with the common pdf given by $f(x) = \frac{1}{\beta}e^{-x/\beta}$, $x > 0$, $\beta > 0$. Define the order statistics $Y_1 \leq Y_2 \leq \ldots \leq Y_4$. Let us denote $U_1 = Y_1$, $U_2 = Y_2 - Y_1$, $\ldots$, $U_4 = Y_4 - Y_3$.

   (a) Show that $U_1, U_2, \ldots, U_4$ are independent and that each U_i has the exponential distribution with mean $(4 - i + 1)^{-1}\beta$, $i = 1, 2, \ldots, 4$.

   (b) Show that $E(Y_k) = \beta\left[\frac{1}{4} + \frac{1}{3} + \cdots + \frac{1}{4-k+1}\right]$ for all $k = 1, \ldots, 3$.

3. Let $X_1, X_2, \ldots, X_5$ be i.i.d. random variables with common probability density function, for $0 < \theta < \infty$,

$$f(x;\theta) = \begin{cases} \frac{1}{2\theta}, & 0 < x < 2\theta \\ 0, & \text{elsewhere} \end{cases}.$$

   (a) Find the pdf of $Y_5 = \max\{X_1, X_2, \ldots, X_5\}$.

   (b) Derive a two-sided 95% confidence interval for θ based on Y_5.

4. Suppose that $X_1, \ldots, X_n$ are i.i.d. $N(\mu, \sigma^2)$ where μ, σ are both unknown with $\mu \in (-\infty, \infty)$, $\sigma \in (0, \infty)$, $n \geq 2$. It is well known that $(\overline{X}, S^2)$ is a complete sufficient statistics of (μ, σ^2).

   (a) Find the minimum variance unbiased estimator of μ.

   (b) Find the minimum variance unbiased estimator of σ^2.

   (c) Find the minimum variance unbiased estimator of $\mu\sigma^2$.

5. Suppose that $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ and that $Cov(X_1, X_2) = -\frac{\sigma_1^2 + \sigma_2^2}{2}$. What is the distribution of $(X_1 + X_2)^2$?

6. The following data show the effect of two soporific drugs (change in hours of sleep) on two groups consisting of 10 patients each:

group	Change in hours of sleep	mean	Standard deviation
1	0.7, −1.6, −0.2, −1.2, −0.1, 3.4, 3.7, 0.8, 0.0, 2.0	0.75	1.79
2	1.9, 0.8, 1.1, 0.1, −0.1, 4.4, 5.5, 1.6, 4.6, 3.4	2.33	2.00

Perform a two-sample t-test for the effect of two soporific drugs:

H0 : The effect of drug 1 = the effect of drug 2

v.s. Ha : The effect of drug 1 is not equal to the effect of drug 2.

7. Let $Y_1, Y_2, ..., Y_n$ be a random sample from a density that has mean μ and variance σ^2.

(a) Show that $\sum_{i=1}^{n} a_i Y_i$ is an unbiased estimator of μ for any sets of constants $a_1, a_2, ..., a_n$ Satisfying $\sum_{i=1}^{n} a_i = 1$.

(b) If $\sum_{i=1}^{n} a_i = 1$, show that the variance of $\left[\sum_{i=1}^{n} a_i Y_i\right]$ is minimized for $a_i = \frac{1}{n}, i = 1, ..., n$.

8. Depression is a significant factor in job performance for police officers. A large police department decided to study the association between marital status and depression. A large group of volunteers answered a questionnaire about their personal lives, and were then assessed for depression on two occasions. The depression score for each volunteer was the average of these two assessments. 4 marital classes were determined: never-married (12), married (34), widowed (8), divorced (36), with final sample sizes indicated in the brackets.

a) Fill in the 3 missing entries in the ANOVA Table below:

Source	Degree of freedom
Marital class	
Error	
total	

b) The police chief wishes to compare the mean levels of depression for the married group versus the mean for the other 3 groups. Write down an appropriate contrast.

c) The police chief also wishes to compare the widowed and divorced groups. Write down an appropriate contrast.

d) The police chief notices that the level of depression in the divorced group is much higher than that in the other 3 groups, and that the next highest level of depression is in the widowed group. The t-value for the contrast is between the divorced and widowed group is $t^*=3.2$. What is the p-value for this contrast (without any multiple comparisons adjustment)?

9. Given the following information:

FULL MODEL

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	0.64378	0.16095	43.88	0.0004
Error	5	0.01834	0.00367		
Corrected Total	9	0.66212			

Number in Model	R-Square	MSE	SSE	Variables in Model
1	0.8546	0.01203	0.09626	X2
1	0.8270	0.01432	0.11456	X4
1	0.6803	0.02646	0.21169	X3
1	0.2759	0.05993	0.47944	X1
2	0.9527	0.00447	0.03131	X2 X4
2	0.9367	0.00598	0.04189	X2 X3
2	0.9046	0.00902	0.06315	X1 X2
2	0.8737	0.01195	0.08362	X1 X4
2	0.8361	0.01550	0.10853	X3 X4
2	0.7289	0.02564	0.17949	X1 X3
3	0.9606	0.00435	0.02611	X2 X3 X4
3	0.9542	0.00506	0.03034	X1 X2 X4
3	0.9370	0.00696	0.04174	X1 X2 X3
3	0.9251	0.00827	0.04959	X1 X3 X4
4	0.9723	0.00367	0.01834	X1 X2 X3 X4

a) Use the information to test H_0: $\beta_2 = 0$ in the model Y = X2

b) Use the information to test H_0: $\beta_3 = \beta_4 = 0$ in the mode Y=X1 X2 X3 X4

c) Perform backward regression. You must justify each step

Table entry for z is the area under the standard normal curve to the left of z.

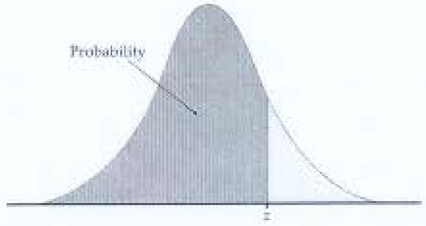

TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

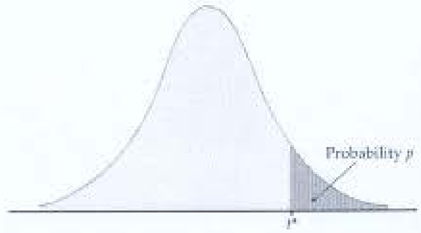

TABLE D

t distribution critical values

	Upper-tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											