The College of Arts & Sciences
Department of Mathematical Sciences
Colloquium

Professor Dmitry Ryabogin

Kent State University
Monday, October 28, 2024
French Hall West, Room 4211
4:00-5:00pm

On the homothety conjecture for convex bodies of flotation on the plane

Let K be a body in $\mathbb{R}^2$, i.e., $K \neq \varnothing$, K is compact, the interior of K is connected, and K is the closure of its interior. For every $\theta \in \mathbb{R}$ and the corresponding unit vector $e(\theta) = (\cos\theta, \sin\theta)$ and for every $t \in \mathbb{R}$, define the half-planes

$$W^+(\theta, t) = \{x : \langle x, e(\theta)\rangle \geq t\} \quad \text{and} \quad W^-(\theta, t) = \{x : \langle x, e(\theta)\rangle \leq t\}.$$

If $0 < \mathcal{D} < 1$, then for every $\theta \in \mathbb{R}$, there is a unique $t(\theta)$ such that

$$\mathbf{vol}_2(W^+(\theta, t(\theta)) \cap K) = \mathcal{D}\,\mathbf{vol}_2(K).$$

The corresponding convex body of flotation $K^{\mathcal{D}}$ is defined as

$$K^{\mathcal{D}} = \bigcap_{\theta\in\mathbb{R}} W^-(\theta, t(\theta)).$$

The body $K^{\mathcal{D}}$ can be viewed as the set of points that stay above the water level when a solid with shape K of uniform density $\mathcal{D}$ floats in any orientation.

We show, in particular, that there is a body different from an ellipse with the property that $K^{\mathcal{D}}$ is homothetic to K.

Refreshments will be served 3:15-3:45 pm in the Faculty Lounge
4118 French Hall West

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