Prof Vilmos Komornik

Université de Strasbourg

Thursday, April 6, 2017
Rm 240 WCharlton Hall
4 – 5 pm

Topological and Fractal Properties of Non-integer Base Expansions

In 1957 A. Rényi generalized the usual integer base expansions to non-integer bases. This led to interesting and deep problems in number theory, combinatorics, set theory, probability, measure and ergodic theory, symbolic dynamics and topology.

Given a base $1 < q \le 2$, following Rényi 1957 we consider expansions of the form

$$x = \sum_{i=1}^{\infty} \frac{c_i}{q^i}$$

with digits c_i belonging to $\{0, 1\}$. In case $q = 2$ of the familiar binary expansions every $x \in [0, 1]$ has an expansion, and this is unique except the dyadic rational numbers that have two expansions.

The case $1 < q < 2$ is radically different: almost every $x \in [0, \frac{1}{q-1}]$ has a continuum of expansions. However, there may still be many numbers having a unique expansion, and the purpose of this talk is to investigate the sets U_q of such "univoque" numbers.

We determine the values of q for which U_q is closed or even a Cantor set. Moreover, we determine the Hausdorff dimension of the sets U_q. It turns out to depend on q in a quite intricate way.

Our survey is mostly based on joint research with P. Erdős, I. Joó, P. Loreti, M. de Vries, D. Kong and W. Li.

Refreshments will be served 3:15 – 3:45 pm in the Faculty & Graduate Student Lounge Rm 4118 French Hall West

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