The McMicken College of Arts & Sciences
Department of Mathematical Sciences

Colloquium

Professor Andrew Lorent
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Thursday, February 28, 2013
Rm 125 WCharlton
3:30 – 4:30 pm

Functions whose symmetric part of gradient agree, the Beltrami equation and Stoilow factorization

We will answer a natural question about the relationship between pairs of functions whose symmetric part of gradient agree. This question is partly motivated by elasticity. Then by studying the geometric meaning of the Beltrami equation we will find out that this question is a special case of the much studied topic of Stoilow factorisation.

We will then state and sketch a rigidity result for pairs of mappings of integrable dilatation whose gradients pointwise deform the unit ball to similar ellipses. This is the first Stoilow factorisation for pairs of mapping where neither is invertible. Our result implies as corollaries a version of the generalized Stoilow decomposition provided by Theorem 5.5.1 of a recent monograph of Astala-Iwaniec-Martin and the two dimensional rigidity result for functions whose symmetric part of gradient agree.

Our main result is the following. Let $u, v \in W^{1,2}(\Omega, \mathbb{R}^2)$ where $\det(Du) > 0$, $\det(Dv) > 0$ a.e. and u is a mapping of integrable dilatation. Suppose for a.e. $z \in \Omega$ we have $Du(z)^T Du(z) = \lambda Dv(z)^T Dv(z)$ for some $\lambda > 0$. Then there exists a meromorphic function ψ and a homeomorphism $w \in W^{1,1}(\Omega : \mathbb{R}^2)$ such that $Du(z) = \mathcal{P}(\psi(w(z)))Dv(z)$ where $\mathcal{P}(a+ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. We show by example that this result is sharp in the sense that there can be no continuous relation between the gradients of Du and Dv on a dense open connected subset of Ω unless one of the mappings is of integrable dilatation.

Refreshments will be served at 2:45 pm in the Faculty & Graduate Student Lounge Room 4118 French Hall West

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