Calculus Contest May 16, 2006.

Name: _____

Show all work. No Calculators. S.S. # _____

Section # _____

1. Find the derivative of $f(x) = x|x|^p$, where $p > 0$ is a real number. The answer should be written as a single formula.

2. (i) Prove that the number $\sqrt{2}$ is irrational.

(ii) Assuming that $\sqrt{2}$ is irrational, prove that $\sqrt{2} + \sqrt{3}$ is irrational.

3. Find the limit

$$\lim_{x \to 0} x\, e^{-\frac{1}{x^2}}.$$

4. Evaluate the integral

$$\int_0^\infty e^{-x} \sin x \, dx.$$

5. Evaluate the sum of the series

$$\Sigma_{n=1}^{\infty} \frac{1}{2n^2 - n}\,.$$

Hint: Write each term of the series as a difference of two (partial fractions!), and then relate this series to that for ln 2.

6. Evaluate the integral

$$\int (x^6 + x^3)\sqrt[3]{x^3 + 2}\,dx.$$

7. Prove that there exists a positive number λ, $\lambda \in (0,1)$, so that

$$\int_0^\pi x^\lambda \sin x \, dx = 3.$$

8. a) Find a strictly positive unbounded function (i.e., with at least one arbitrarily large peak) on the real line with finite area. Feel free to splice functions together.
b) Find a strictly positive function on the real line with infinite area but finite volume of revolution.
c) Find a strictly positive function on the real line with infinite surface area but finite volume.