Calculus Contest May 15, 2007.

Name: ____________________

Show all work. No Calculators.

UCID # ____________________

Section # ____________________

1. Find the derivative of $f(x) = x|x|^p$, where $p > 0$ is a real number. The answer should be written as a single formula. Is this function even or odd?

2. Calculate $\int_0^{\pi/2} \sqrt{1+\sin x}\, dx$.

Hint: $1 = \sin^2 \dfrac{x}{2} + \cos^2 \dfrac{x}{2}$. (Or do it your way.)

3. For any positive integer n

$$n^2 = n + n + \ldots + n,$$

where the sum on the right has n terms. Differentiating both sides,

$$2n = 1 + 1 + \ldots + 1,$$

i.e.

$$2n = n.$$

Dividing by n,

$$2 = 1.$$

Is there anything wrong with this argument? Explain.

4. Evaluate the limit

$$\lim_{x \to -2} \frac{|x+1| - 1}{4 - x^2}.$$

Justify your answer.

5. (i) Let $g(x) = \int_0^x (x-t)f(t)\,dt$. Show that $g''(x) = f(x)$.

Hint: Break the integral into two pieces. (Or do it your way.)

(ii) Let $G(x) = \dfrac{1}{(n-1)!}\int_0^x (x-t)^{n-1} f(t)\,dt$. Evaluate the n-th derivative $G^{(n)}(x)$.

6. Evaluate the integral

$$\int x^3 \sin(x^2)\,dx.$$

7. (i) Does the series $\Sigma_{n=2}^{\infty} \dfrac{(-1)^n}{n}$ converge? Explain.

(ii) Does the series $\Sigma_{n=2}^{\infty} \dfrac{(-1)^n}{n} \left[\dfrac{(-1)^n}{\ln n} + 1\right]$ converge? Explain.

(iii) State the limit comparison test. Will the test remain valid if one no longer requires that both series have positive terms? Explain.