Name:____________________ M#:________ Instructor:________________

Instructions: This exam has seven problems on seven pages. Show all your work, expressing yourself in clear and concise manner. Do as many problems as you can, but be advised that a complete solution to a problem may be worth more than several partial ones. Use the backs of the exam pages for work, if necessary. No calculators of any kind are allowed.

1. Suppose that f'' is continuous on $[0, \pi]$ and that

$$\int_0^\pi [f(x) + f''(x)] \sin x \, dx = 2.$$

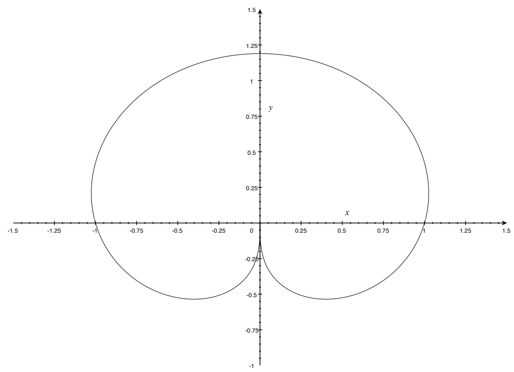
Given that $f(\pi) = 3$, compute $f(0)$.

2. Let $A(t)$ be the area under the curve $y = \sin(x^2), 0 \le x \le t$. Let $B(t)$ be the area of the triangle with the vertices $(0,0)$, $(t, \sin(t^2))$, and $(t, 0)$. Find $\lim_{t \to 0^+} \dfrac{A(t)}{B(t)}$.

3. A right triangle whose three sides have lengths 3, 4, and 5 ft is rotated about its hypotenuse. Compute the area of the resulting surface of revolution.

4. Let $f(x) = \displaystyle\int_1^x \frac{\ln t}{1+t}\,dt$ for $x > 0$. Find a formula for $f(x) + f(\frac{1}{x})$ that does not involve integrals.

5. Find the area of the region enclosed by the polar curve $r(\theta) = (1 + \sin\theta)^{1/4}$, $0 \le \theta \le 2\pi$. (Hint for your integral: $1 = \sin^2(\theta/2) + \cos^2(\theta/2)$.)



6. (a) Find the Maclaurin series for $\cos^2(x)$ and state its radius of convergence. (Hint: no need to square a power series.)

(b) Use the fact that

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi} \quad \text{and} \quad \int_{-\infty}^{\infty} x^{2n} e^{-x^2}\,dx = \frac{(2n-1)!}{2^{2n-1}(n-1)!}\sqrt{\pi},\ n \geq 1,$$

together with your result for part (a), to show that

$$\int_{-\infty}^{\infty} e^{-x^2}\cos^2(x)\,dx = \frac{(1+e)\sqrt{\pi}}{2e}.$$

7. Let f be a continuous function on the interval $[0,1]$ such that $\int_0^1 f(t)\,dt = 0$. Show that

$$\int_0^1 e^{af(t)}dt \geq 1$$

for any real number a.

(Hint: treat the left-hand side of this inequality as a function of a and examine its derivative(s), in particular at zero. Assume that you can differentiate under the integral sign.)