Name:_____   M#:_____   Instructor:_____

> **Instructions**: This exam has seven problems on seven pages. Show all your work, expressing yourself in clear and concise manner. Do as many problems as you can, but be advised that a complete solution to a problem may be worth more than several partial ones. Use the backs of the exam pages for work, if necessary. No calculators of any kind are allowed.

1. Calculate $\displaystyle\int_0^{\pi/2} \sqrt{1 + \sin x}\, dx$.

   Hint: Use $1 = \cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}$. (Or do it your way.)

2. Integrate.

$$\int \frac{x \arctan x}{(1+x^2)^2} \, dx$$

3. Suppose you're standing on top of a light house so your eyes are 100 feet above the level of a calm sea. You look out to where the sea and sky meet. About how far is that horizon? Give your answer in miles and provide some estimate of your accuracy. You can assume that the Earth is a sphere with radius 4000 miles. There 5280 feet in a mile.

4. For any positive integer n

$$n^2 = n + n + \cdots + n \,,$$

where the sum on the right has n terms. Differentiating both sides with respect to n we obtain

$$2n = 1 + 1 + \cdots + 1 \,,$$

or

$$2n = n \,.$$

Dividing by n,

$$2 = 1 \,.$$

Is there anything wrong with this argument? Explain.

5. At what points on the graph of $y = x^2 e^{-x}$ does the tangent line have y-intercept equal to 0?

6. A particle is moving in the xy-plane along the polar curve $r = 2\sin(3\theta)$, $0 \le \theta \le \pi$. Its polar angle, θ, is a function of time, t. When the particle is at the point $(0, -1)$, θ is changing at a rate $\frac{d\theta}{dt} = -3$. How fast is the x-coordinate of the particle changing at that point?

7. Let g be a continuous function on the interval $[0, 1]$. Assume that $\int_0^1 g(x)\,dx = 0$. Show that

$$\int_0^1 e^{g(x)} g(x)\,dx \geq 0.$$