U.C. MATH BOWL 2019

LEVEL I— Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. Two lines with slopes m and n, with $m > n > 0$, intersect at the origin. The line $y = x$ bisects the angle between the two lines. What is the value of mn?

2. At a neighborhood potluck 3 Singhs, 3 Zhangs, and 2 Slavins sit at random around a circular table. What is the chance that each person is sitting next to at least one person from another family?

3. Suppose a and b are positive numbers with the amazing property that

$$\log_a(b) - \log_b(a) = 3.$$

   What is $(\log_a(b))^2 + (\log_b(a))^2$?

4. Two numbers a and b are such that the equation $\sin x + a = b\,x$ has exactly two solutions. Prove that the system

$$\sin x + a = b\,x, \quad \cos x = b$$

   has at least one solution. Hint: Draw a picture!

5. Suppose f is a continuously differentiable function, that $f'(2) = \pi$ and that $f(2) = -4$. Find the limit

$$\lim_{x \to 2} \frac{f(x) + 4}{\sqrt{x} - \sqrt{2}}.$$

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LEVEL I — Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. The three-digit numbers $C99$, $A6A$, $BC7$ and $B91$ form an arithmetic sequence in this order.

   Capital letters represent digits so in this notation XYZ is the number $100X + 10Y + Z$.

   Recall that in an arithmetic sequence the difference between one term and the next is a constant. What is the value of $A2 + B2 + C2$?

2. What is the sum of all the integer values of x for which $|3x - 3| < 13$?

3. If n is a positive integer write $s(n)$ for the sum of n's digits. So, for example, $s(543) = 5+4+3 = 12$.

   (a) What is $s(1) + s(2) + s(3) + \cdots + s(9)$?

   (b) What is $s(1) + s(2) + s(3) + \cdots + s(10^4)$? (hint: how does part (a) help here?)

4. If
$$\sin(x) + \sin^2(x) + \sin^3(x) + \cdots = 4$$
   what could be the value of
$$\cos(x) + \cos^2(x) + \cos^3(x) + \cdots ?$$

5. Find the area of the region bounded by the graphs of $y = x^2$, $y = 2(x-4)^2$, and $y = 0$.