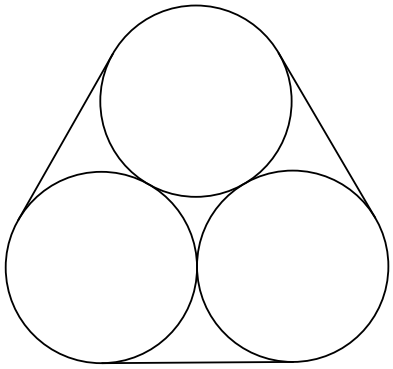
U.C. MATH BOWL 2019

LEVEL II — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. The figure shows three pipes, in cross-section, bound together by a strong band. If the pipes are each 1 foot in diameter how long is the band?



2. How many three digit integers (i.e. from 100 to 999) have the property that none of their adjacent digits are the same?

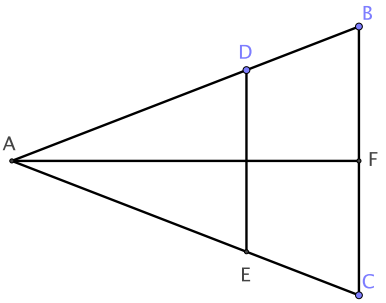
3. If n is a positive integer write $s(n)$ for the sum of n's digits. So, for example, $s(543) = 5+4+3 = 12$.

   (a) What is $s(1) + s(2) + \cdots + s(9)$?

   (b) What is $s(1) + s(2) + s(3) + \cdots + s(1000)$? (hint: how does part (a) help here?)

4. A and B are positive, two-digit numbers so that $AB = 9009$. What is $A + B$?

5. Consult the figure. Points D and E are situated so that both $\triangle ABC$ and $\triangle ADE$ are isosceles with $AB = AC$ and $DE \parallel CB$. F is the midpoint of BC.



Suppose that the area and the perimeter of $\triangle ADE$ and trapezoid $DBCE$ are equal.

Calculate $|FB|/|AB|$.

U.C. MATH BOWL 2019

LEVEL II — Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. Three people stand in a line facing the same direction. Person C is at the end of line and can see A and B. Person B is in the middle and can only see A. And, A can see no one.

   A puzzle master shows the people that she has 3 blue hats and 2 red hats. With their eyes closed, she puts a one of these hats on each person's head and then invites them to open their eyes.

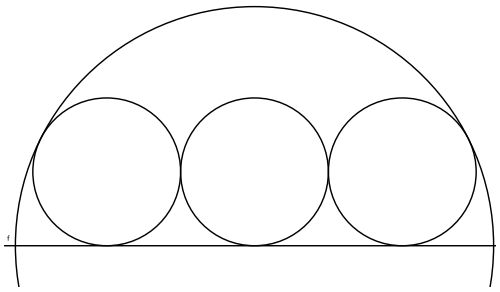
   Person C says, "I can't tell what color hat is on my head."

   Person B says, "I can't tell what color hat is on my head."

   Person A says, "I know what color hat is on my head."

   How does person A know, and what color is the hat?

2. The centers of three circles of radius 1 lie on a line; A semicircle is circumscribed around these smaller circles so that its diameter is tangent to all three of them and so that it is tangent to the outer two small circles. What is the radius of the semicircle?



3. Suppose that $f(n)$ is defined by

$$f(n) = \begin{cases} n-1 & \text{if } n \text{ is even} \\ n^2-1 & \text{if } n \text{ is odd} \end{cases}$$

   Find all the numbers n so that $f(f(n)) = 8$.

4. In a certain country, every 20th mathematician is a writer, while every 60th writer is a mathematician. Are there more writers or mathematicians in the country? How many times more?

5. How many non-congruent triangles with sides of integer lengths and with perimeter 7 are there?