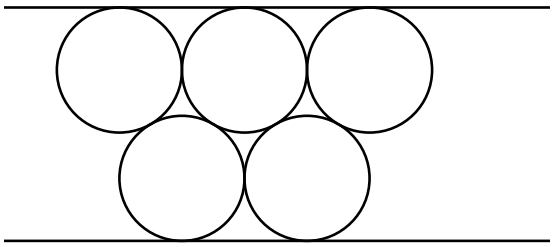
U.C. MATH BOWL 2018

LEVEL I — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. Circles with diameter 2 are packed together in the pattern shown in the figure so that they just touch each other.



How far apart are the parallel lines that just touch the circles?

2. These statements were all made on the same day:

   (a) Yesterday was Monday

   (b) Tomorrow is Friday

   (c) The day after tomorrow will be Friday

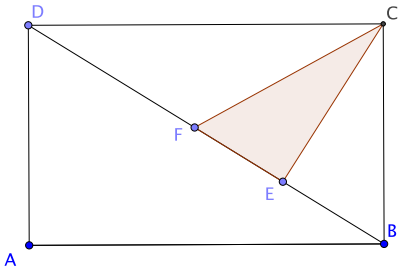
   (d) Tomorrow will be Saturday

   (e) The day before yesterday was Tuesday

   An additional interesting fact is this: *If you knew how many of the statements were correct, you could determine on which day of the week the statements were made.*

   On what day of the week were the statements made?

3. In the figure, the area of rectangle $ABCD$ is 100 square units.



   Arranged as shown, points E and F are selected on BD so that $2(|BE| + |FD|) = 3|EF|$. What is the area of triangle $\triangle CEF$?

4. Suppose that $f(x) = x\sqrt{1+x^2}$ and that $g(x) = f \circ f \circ f(x) = f(f(f(x)))$. What is $g'(1)$?

5. The function $f(x) = \cos(x) + 3\sin(2x)$ is defined for all x. Find all the numbers x so that $f'(x) = 0$.

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LEVEL I — Session 2

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Have Fun!

1. What is the the remainder when 1000^{2018} is divided by 2018?

2. Without approximating the square roots (or using a calculator) explain why
$$\sqrt{7} < \frac{5}{3}\sqrt{5}.$$

3. The function $f(x) = \cos(x) + 3\sin(2x)$ is defined and has a derivative at every x. What's more, $f'(x) = 0$ exactly when $\sin(x) = 2/3$ or $\sin(x) = -3/4$. Find the absolute maximum value of $f(x)$.

4. Find the value of
$$\lim_{n\to\infty} \sum_{k=1}^{k=n} \frac{k}{k^2+n^2}.$$

5. The curve defined (implicitly) by the equation $y^3 + x^3 = 3xy + 3$ passes through the point $(2, 1)$. Where else does the tangent to the curve at this point meet the curve?