

U.C. MATH BOWL 2017

LEVEL I — Session 1

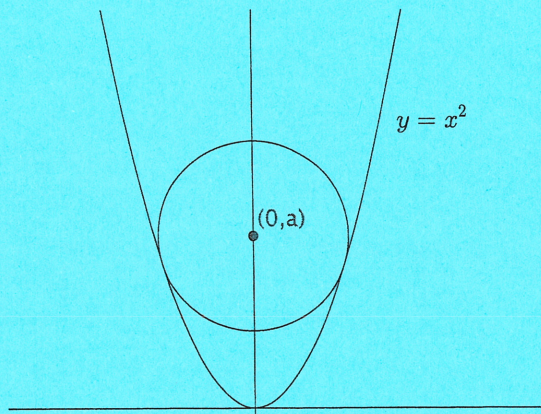
Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. Recall that $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$. Which is bigger

$$\sqrt[999]{999!} \quad \text{or} \quad \sqrt[1000]{1000!}?$$

2. The figure shows the graph of the parabola $y = x^2$ and a circle with center $(0, a)$ that is tangent to the parabola.



For each $a > 0$ determine the radius of the circle with center $(0, a)$ that is tangent to the parabola.

3. The line ℓ is tangent at $(-1, -2)$ to a circle centered at $(0, 0)$. What is the x intercept of ℓ ?
4. A certain type of toy brick comes in two sizes: 2 cm by 4 cm, and 2 cm by 2 cm. The 2 x 4 bricks cost 11 cents, the 2 x 2 bricks cost 5 cents. If you are restricted to a maximum of 200 of each type of brick, what is the largest area you can cover with \$16 worth of bricks? How many of each type of brick would be used?
5. Find constants a and b so that, for $0 < x < \pi/12$,

$$\frac{\sin(9x)}{\cos(3x)} + \frac{\cos(9x)}{\sin(3x)} = a \cot(bx)$$