U.C. MATH BOWL 2017

LEVEL I — Session 2

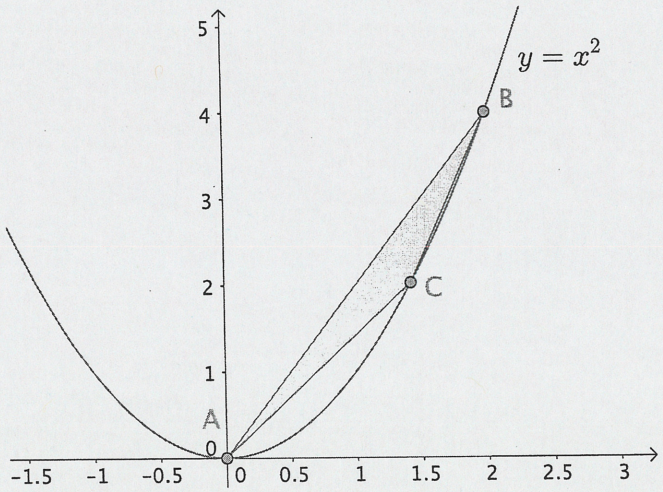
Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. The polynomial $2x^3 + 3x^2 + 5x + 2$ has two roots x_1 and x_2 that are complex numbers — they are not real numbers. What is the sum $x_1 + x_2$?

2. Evaluate the limit
$$\lim_{x \to \infty} \frac{1}{x^4} \int_0^{x^2} \frac{t^4}{1+t^3}\, dt$$

3. Find the point C on the graph of $y = x^2$ between the points $A(0,0)$ and $B(2,4)$ so that the area of $\triangle ABC$ is as large as possible.



4. Consider the array of consecutive odd numbers (each row beyond the first contains 2 more numbers than the previous row):

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 3 & 5 & 7 & & \\ & 9 & 11 & 13 & 15 & 17 & \\ 19 & 21 & 23 & 25 & 27 & 29 & 31 \\ & & & \vdots & & & \end{array}$$

In what row will the number 2017 appear?

5. The arithmetic sequence $a_1, a_2, a_3, \ldots, a_n$ has a common difference of 10 and the sum $a_1 + a_2 + \cdots + a_{100} = 2017$. Determine the value of the sum of even terms:

$$S = a_2 + a_4 + \cdots + a_{100}.$$