U.C. MATH BOWL 2017

LEVEL II — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

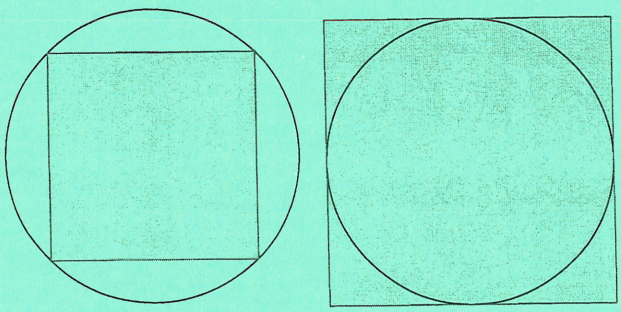
Have Fun!

1. If 1/5 and 7/15 are the first and fifth terms of an arithmetic sequence, what is the sum the second, third, and fourth terms?

2. Suppose you pick an integer at random between 1 and 100 inclusive. What is the chance it is the sum of exactly two square numbers?

3. The calculation
$$(3+\sqrt{2})^2 = 11+6\sqrt{2}$$
shows that the number $11+6\sqrt{2}$ has a square root with the form $a+b\sqrt{2}$ where a and b are integers. Show that the number
$$19+6\sqrt{2}$$
also has a square root of this form.

4. What's a better fit: a round peg in a square hole or a square peg in a round hole?



   (a) Find the ratio of the area of a circle to the area of the square in which it is inscribed.

   (b) Find the area of a square to the area of the circle in which it is inscribed.

   (c) Which of these ratios is larger?

5. Find a two digit number N with the property that 3 times the number minus 4 times the number with the digits reversed is 19. For example, if $N = 27$ then the number with the digits reversed is 72.