# U.C. MATH BOWL 2017

LEVEL III — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. At William Diamond High School, 75% of all athletes are honor students, but, only 15% of all honor students are athletes. If 18 athletes are not honor students, how many honor students are not athletes?

Answer: 306

Solution: Since 75% of all athletes are honor students, 25% are not honor students. If the number of athletes who are not honor students is 18 and this number represents 25% of all athletes, then, there are $18 \times 4 = 72$ athletes. Thus, there must be $72 - 18 = 54$ athletes who are also honor students. Since 15% of all honor students are athletes, these 54 athletes who are also honor students represent 15% of all honor students. Letting $T$ be the total number of honor students we have: $0.15 \times T = 54$. Solving for $T$we find $T = 360$. So, there must be $360 - 54 = 306$ students who are honor students but not athletes.

2. If it were two hours later, it would be half as long until midnight as it would be if it were one hour later. What time is it?

Answer: 9 PM

Solution: If $x$ is the time, measured in hours since noon, we are told

$$12 - (x + 2) = (1/2)(12 - (x + 1)).$$

That tells us $x = 9$.

3. Exactly 11 toothpicks can be arranged to form the following incorrect statement involving Roman numerals.

$$VI + II = V.$$

Can you move just one toothpick to make a correct statement? Can you find more than one way to do this?

Some possible solutions:

$$VI - II = \mathbf{I}V$$

$$VI - II \neq V$$

$$VI\mathbf{I} - II = V$$

$$II + II = \mathbf{I}V$$

And here's a couple more that are slightly suspect but acceptable.

$$VI + II > V$$

$$V + I - I = V$$

4. At a party, the guests discover that among any group of five of them, at least two of them share a birthday. What is the greatest possible number of distinct birthdays among all the guests? Based only on this information, how big of a group would you need to be sure that at least three of them share a birthday?

Answer: There are no more than 4 distinct birthdays; you would need to pick nine people.

Solution: If there were five distinct birthdays within the group, then it would be possible to select those five people and have no duplicates. Thus, there must be no more than four distinct birthdays within the group. With four distinct birthdays, you could pick eight different people and have two people with each birthday. Once you add a ninth, that person's birthday will overlap with one of the other pairs, so there will be at least three people who share the same birthday among those nine.

5. In the football game Saturday the home team only scored points by touchdowns (worth 7 points) and field goals (worth 3 points). What is the largest number of points that the team could not have scored?

Answer: 11

Solution: 12=4(3); 13=7+2(3); 14=2(7); Any larger number can be obtained from one of these by adding some additional field goals. We can see that 11 points can not be scored by noting that it can not be made by just scoring field goals as it is not divisible by 3. There

can have been at most one touchdown in which case the remaining 4 points could not be made up of field goals.