U.C. MATH BOWL 2017

LEVEL II — Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. There is an imaginary island called the Island of Knights and Knaves. On this island, there are people called knights, who always tell the truth, and people called knaves, who always lie. The two types are indistinguishable by sight. On a vacation to this island, you meet two inhabitants: Bill and Bob. Bob says "We are both the same kind, either both knights or both knaves", but Bill says "We are both different kinds. One of us is a knave and the other is a knight" Who is the knight and who is the knave?

Answer: Bob is a knave and Bill is a knight.

Solution: Suppose Bob is a knight. Then he is telling the truth, and this means Bill is a knight, and Bill is telling the truth. But if Bill is a knight, Bill actually telling a lie by saying they are different kinds! This cant be!

Suppose Bob is knave. When he says they are both the same kind, he is lying, which means that Bill must be a knight. If Bill is a knight, he is telling the truth when he says they are different kinds.

2. In the football game Saturday the home team only scored points by touchdowns (worth 7 points) and field goals (worth 3 points). What is the largest number of points that the team could not have scored?

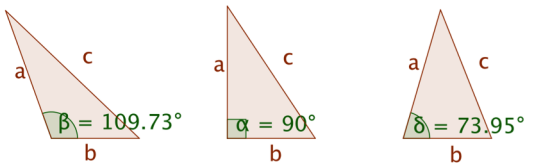
Answer: 11

Solution: Observe that $12 = 4(3)$; $13 = 7+2(3)$; $14 = 2(7)$. Any larger number can be obtained from one of these by adding some additional field goals. We can see that 11 points can not be scored by noting that it can not be made by just scoring field goals as it is not divisible by 3. There can have been at most one touchdown in which case the remaining 4 points could not be made up of field goals.

3. Two sides of a triangle have lengths 2 and 3 and the third side has length L. What could L be if the triangle has only acute angles?

Answer: $\sqrt{5} < L < \sqrt{13}$

Solution: In a triangle with sides a, b, c the angle opposite the side c is right if and only if $a^2 + b^2 = c^2$; It is larger than $90°$ is and only if $a^2 + b^2 < c^2$; it is acute if and only if $a^2 + b^2 > c^2$.



$c^2 > a^2 + b^2$ $\quad$ $c^2 = a^2 + b^2$ $\quad$ $c^2 < a^2 + b^2$

We can express that every angle in our triangle is acute with the conditions

$$2^2 + 3^2 > L^2$$
$$2^2 + L^2 > 3^2$$
$$3^2 + L^2 > 2^2$$

The last inequality is true if the second one is. The others we can summarize as

$$3^2 - 2^2 < L^2 < 3^2 + 2^2.$$

4. Determine the value of he product of all roots of the polynomial $3x^5 + 12x^4 - 7x^3 - 5x^2 + 6$ times the sum of these roots.

The roots are the same for the polynomial $x^5 + 4x^4 + (7/3)x^3 - (5/3)x^2 + 2$. Observing the constant term and the coefficient of x^4 we see that for this polynomial the sum of the roots is -4 and the product of the roots is -2. The product of these numbers is 8.

5. Sam and Terry rake a yard for 40 minutes, after which Lou joins them, and they finish 20 minutes later. If Lou had not helped, it would have taken them 50 more minutes to rake the yard. How long would it have taken Lou to rake the yard alone?

Answer: 1 hour.

Solution: S and T can rake the yard once in 90 minutes. S and T and L rake 5/9 of a yard in 20 minutes. So in 180 minutes S and T could rake the yard twice and S, T, and L could rake it 5 times. So L could rake the yard by himself 3 times in 180 minutes or once in 60 minutes.