

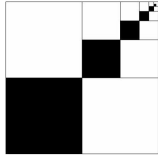
U.C. MATH BOWL 2020

LEVEL II — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

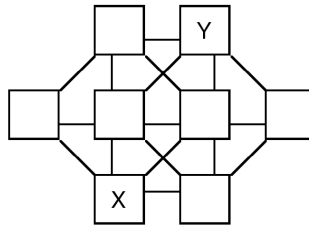
Have Fun!

1. Assuming that the indicated pattern continues on forever what fraction of the original square is colored



black?

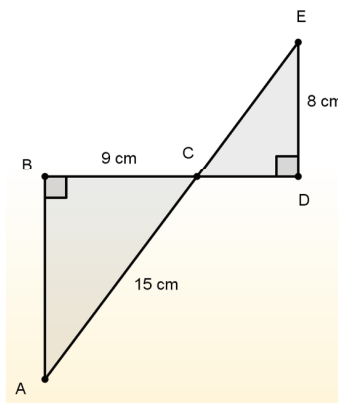
2. In the diagram shown, the boxes are to be filled with the digits 1 through 8 (each used exactly once). No two boxes connected directly by a line segment can contain consecutive digits. What is the sum of the digits in the boxes marked X and Y?



3. You sit at a sprawling table with a pile of thousands of quarters in front of you, but you don't know exactly how many. You have a blindfold on, so you cannot see the quarters, but you do know that exactly 20 quarters are tails-side-up, and the rest are heads-up. You can move the quarters and flip them over as much as you want, but remember, you cannot see what you are doing. Though you can feel the quarters, you cannot determine which side is heads and which side is tails just by touch. How do you separate the quarters into two groups that have the same number of tails-side-up quarters in them?
4. Suppose a and b are positive numbers with $a < b$. Arrange the following in increasing order:

$$\frac{b+1}{a+1}, \frac{b+1}{a}, \frac{b}{a}, \frac{b}{a+2}, \frac{b}{a+1}$$

5. In the figure line BD intersects AE at the point C . The length of BC is 9 cm, then length of AC is 15 cm and the length of ED is 8 cm. Find the combined area of triangle ABC and triangle DEC .



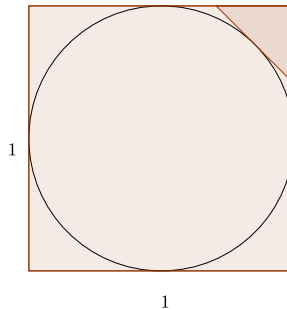
U.C. MATH BOWL 2020

LEVEL II — Session 2

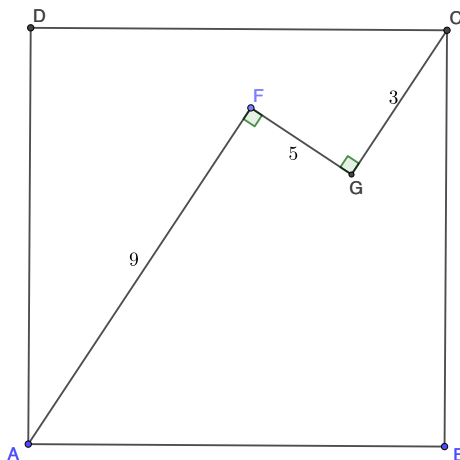
Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. A circle is inscribed in a square with side length 1. In each corner of the square there is a right isosceles triangle that shares a vertex angle with the square and has hypotenuse tangent to the circle. What is the area of one of these triangles?



2. A bag originally contained 12 marbles in some combination of red, green, and blue. After 2 marbles are removed from the bag the probabilities of drawing a marble of different colors satisfy: $P(\text{draw green}) < P(\text{draw red}) < P(\text{draw blue})$.
Is it possible that the original bag contained more green marbles than blue? What about more green marbles than red marbles?
3. Suppose that $O(0, 0)$, $P(a, b)$ and $Q(c, d)$ are three points with integer coordinates between 0 and 2 inclusive. So $0 \leq a, b, c, d \leq 2$. In how many ways can you pick a, b, c, d so that $\triangle OPQ$ is a right triangle with one of its angles measuring 90° ?
4. In the figure, $ABCD$ is a square and points F and G lie inside the square so that $|AF| = 9$, $|CG| = 3$, $|FG| = 5$ and $\overline{AF} \perp \overline{FG}$ and $\overline{FG} \perp \overline{CG}$. Find the area of square $ABCD$.



5. A school has 40 teachers and each teacher teaches 4 classes. Each class has 30 students and 1 teacher. Each student takes 5 classes. How many students are there at the school?