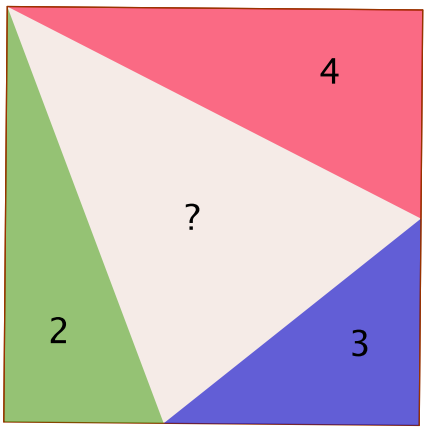
U.C. MATH BOWL 2020

LEVEL III — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. The big shape is a square. Can you replace the ? with a number so that the picture makes sense? Explain! Can you say how long some of the sides of the triangles are?



This is an open ended question. Responses may assume the square is integer sided and consider total areas that are prefect squares. There's other approaches based on arguing about the lengths of the sides of the triangles.

2. You go to a shop and buy three bags of candy. One has chocolates in it, one has licorice in it and the third has some of each. In an effort to be helpful the shopkeeper labels the bags as C, L, and CL but then admits that each bag is mislabeled. Explain how you can examine just one candy from one bag and correctly determine the contents of each of the three bags.

The bag labelled CL contains only C or only L because it is mislabeled. Examine one candy from this bag. If it is C then the bag contains only C. The bag labeled L cannot contain L and it doesn't contain C so it must contain CL. That means that the third bag that is labeled C must contain L. Similar reasoning applies it the bag labeled CL contains only L.

3. Suppose a and b are positive numbers with $a < b$. Arrange the following in increasing order:

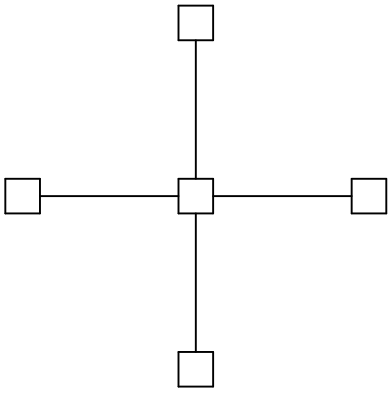
$$\frac{b+1}{a}, \frac{b}{a}, \frac{b}{a+2}, \frac{b}{a+1}$$

We have : $\frac{b}{a+2} < \frac{b}{a+1} < \frac{b}{a} < \frac{b+1}{a}$ based on same number of parts and same sized parts comparisons.

Many teams take the implicit hint that these inequalities hold for all a and b, select explicit values (1, 2) and calculate numerically. This isn't wrong but not as good as could be done.

4. For a set of 5 positive integers the mode is 1, the mean is 4, and the median is 5. What are these 5 numbers?

5. Using the numbers $2, 3, 4, 5, 6, 7$ each at most once can you fill in the boxes so the sum of all the numbers in boxes in each line is the same? How many ways can you do it?



U.C. MATH BOWL 2020

LEVEL III — Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. How many integers from 1 to 100 have the property that the sum of their digits is divisible by 2?

Taking the 1-digit numbers first (0-9), all of the even numbers have digits that are divisible by 2, so four total. The sum of digits of a 2-digit number will be divisible by 2 if both digits are even or if both digits are odd. This means that if the digit in the 10s place is odd, it must be an odd number; if the digit in the 10s place is even, it must be an even number (e.g., 11, 13, 15, 17, 19 work, as do 20, 22, 24, 26, 28). There are five numbers within each decade for which the sum of the two digits is divisible by 2, so 45 total. Because 100 does not work, there are 49 numbers in all.

2. Suppose that in the square shown below each box is filled with a positive number in such a way that the product of the numbers in each row, each column, and on each long diagonal are the same. A few numbers are shown. What number is in the lower-right corner?

	15	3
	9	
		?

Suppose the number in the lower-right is Y and in the upper-left is X.

X	15	3
	9	
		Y

Then $45X = 9XY$. So $Y = 5$.

3. The Titanic struck an iceberg at 11:40 PM on April 14, 1912 causing it to sink at 2:20 AM on April 15th. There 5280 feet in a mile.

   (a) How many seconds elapsed between hitting the berg and sinking?

   (b) The Titanic was 882.75 feet long. How many inches is that?

   (c) The Titanic had a top speed of 28 miles per hour. At that speed approximately how long would it take the Titanic to pass by?

2 hours 40 minutes is 9600 seconds

882.75 feet is 9583 inches.

28 mph is about 41 feet per second. At just less than 883 feet it would take $883/41 \approx 20$ seconds

4. Out of 500 students at Diamond Middle School 200 are in the drama club 225 are in the physics club and 200 are in neither club. How many students are in both the physics and drama club?

Of course this uses $|A \cup B| = |A| + |B| - |A \cap B|$ in some form or another. Some good approaches without using this fact involve solving some linear equations.

5. Find positive integers A, B, C so that

$$A + \frac{1}{B + \frac{1}{C}} = 7.4$$

Since $1/(B + 1/C)$ must be less than 1 (because $B1$ and $1/C > 0$), we must have $A = 7$. And, if $A = 7$, then $1/(B + 1/C) = 4/10$. In which case, taking reciprocals, $B + 1/C = 10/4 = 5/2 = 2.5$ The same reasoning ($1/C < 1$) applies and shows that $B = 2$. Knowing that, we learn $1/C = 1/2$ so $C = 2$.