U.C. MATH BOWL 2021
LEVEL II

There are 7 questions for you work on in this Math Bowl. Each is printed on a separate page.

Write your school, team number, and the names of the team members on the first page.

Write your school and team number on each question's page.

All your work and answers to each question should go on that question's page (or you can use extra pages if you need more room). Please only put the answer to one question on each page.

Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. Can the sum of four consecutive integers be ever divisible by 4 (i.e. a multiple of 4)? Explain.

2. Fifteen children have 100 lollipops among them. Can you explain why at least two children have the same number of lollipops?

3. Which is greater, the sum of all even numbers from 0 to 1000 or the sum of all odd numbers from 1 to 999? By how much? (Hint: you shouldn't use a calculator.)

4. Given a convex quadrilateral, which is bigger the sum of the lengths of its sides or the sum of the lengths of its diagonals? What if the quadrilateral isn't convex? (Convex means that both the diagonal are inside the quadrilateral.)

5.

In the picture above, the same symbols represent the same numbers, and the numbers at the end of each row and column represent the sum of all numbers in that row or column. What goes into the question mark?

6. Find all the numbers k for which $p(x) = x^3 - kx^2 + kx + 2$ has $x - 2$ as a factor.

7. A right triangle has area 36 and perimeter P. How long is the triangle's hypotenuse h? (Your answer will depend on the perimeter P.)