U.C. MATH BOWL 2024

LEVEL III — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. If A and B are integers that satisfy $10 < A < 20$ and $-6 < B < 8$ how big, and how small, can $A - B$ be?

Answer: Smallest: 4 and Biggest:24

Some people treat the inequalities as weak — as saying $10 \leq A \leq 20$ — and this mistake leads to the answers small: $10 - 8$ and big: $20 - (-6)$.

If $10 < A < 20$ and $-8 < -B < 6$ then, taking the largest values A and $-B$ can have we get $A + (-B) \leq 19 + 5$ while, taking A and $-B$ as small as possible we find $A + (-B) \geq 11 - 7$.

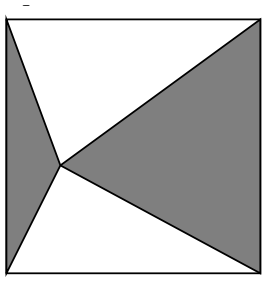
2. Using the numbers $1, 2, \cdots, 9$ each exactly once fill in the 3×3 grid so that the sums of the row and columns are the numbers indicated in the margins of the table.

			15
			10
			20
6	21	18	+

$1, 2, 3$ must appear in the first column as that is the only way to get a sum of 6. 3 must appear in the last row so that the sum of the row can sum to 20 using $9, 8$. If 2 appears in the second row there's now combination of two numbers from 4,5,6,7 that would yield a row sum of 10 in the second row. So 1 appears in the second row and 2 appears in the first row. ...

2	7	6	15
1	5	4	10
3	9	8	20
6	21	18	+

3. The figure shows a rectangle with two shaded triangular regions with a common vertex and whose sides are sides of the rectangle. What fraction of the area of the rectangle is shaded? How do you know?



Ans: 1/2. If the sides of the triangles that are sides of the rectangle are the heights of the rectangle with length h, then the sum of that altitudes a_1, a_2 of these rectangles over those sides adds to the width w of the rectangle. So

$$(1/2)ha_1 + (1/2)ha_2 = (1/2)hw.$$

4. You come across a flock of birds and ask their fearless leader how many birds there are? The fearless leader says "Us, another flock of our size, a flock of half our size, a flock of quarter of our size, and you would make 100 in all." How many birds are there in the flock?

$f + f + f/2 + f/4 + 1 = 100$. So $f = 99(4/11) = 36$.

5. People are walking their dogs in a small park. In all there are 40 heads in the park and 124 legs. How many people and dogs are there?

If there are P people and D dogs the information provided says

$$P + D = 40$$
$$2P + 4D = 124$$
.

That means $2D = 44$ so $D = 22$ and then $P = 18$.

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LEVEL III — Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. As shown in the first row, add operation signs $+ - \times \div$ and parentheses as needed to make a true equation of each of the rows.

$$\begin{array}{ccccccc} 5 & -(5 & \div 5) & -(5 & \div 5)) & = 3 \\ 5 & 5 & 5 & 5 & 5 & = 5 \\ 5 & 5 & 5 & 5 & 5 & = 6 \\ 5 & 5 & 5 & 5 & 5 & = 30 \\ & 5 & 5 & 5 & 5 & = 55 \\ 5 & 5 & 5 & 5 & 5 & = 55 \end{array}$$

Many ways to do most of these, here's one at least:

(a) $5 \times (5/5) \times (5/5) = 5$

(b) $5 + (5/5) \times (5/5) = 6$

(c) $(5 * 5) + (5 * 5/5) = 30$

(d) $5 * (5 + 5) + 5 = 55$

(e) $5 * (5 + 5 + (5/5)) = 55$

2. Fill in the squares of the grid with 9 consecutive integers so that the products of the rows and columns are the numbers indicated in the margin of the table.

			0
			42
			20
0	12	-15	*

One of the numbers must be 0. And one must be negative and it must be in the same row as the 0. Since 7 is a prime factor of 42, one of the numbers you use must be 7. This tells you that the consecutive numbers to use are

$$-1, 0, 1, \cdots, 7.$$

0	6	-1	0
7	2	3	42
4	1	5	20
0	12	-15	$\times$

3. Peter was 10 the day before yesterday. Next year, he'll be 13. Explain how this is possible.

If today is 1 Jan 2024 then two days ago was 30 Dec 2023 and Peter was 10 years old. If his birthday is 31 Dec, then he turned 11 yesterday. This year, on 31 Dec 2024 he'll turn 12. And next year, in 2025, he'll be 13 on 31 Dec.

4. What is the percentage change in the area of a rectangle if one of its sides is increased by 20%, while the other side is decreased by 20%?

$(1 + 0.2) \times (1 - 0.2) = 0.96$ so the area is decreased by 4%.

5. Can you divide the numbers from 1 to 10 into two groups so the sums of the numbers in the two groups are the same? Show how to do this or explain why it can't be done.

It can't be done. If the sum of the numbers in each of the groups is S then

$$2S = 1 + 2 + 3 + \cdots + 10 = 55$$

and that is impossible since 55 is odd.

While the above is the "standard" answer to the question one that kids have come up with in the past, that seems even better, is to say that there's an odd number of odd numbers $(1, 3, 5, 7, 9)$ so if we put them into two groups one group will get an odd number of them and one an even number of them. Adding even numbers to the groups in any way won't change the fact that one group will have an odd sum and the other an even sum — so the sums of the groups can't be equal.