U.C. MATH BOWL 2024

LEVEL I— Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. Suppose that $x + 1/x = 1$. What is $x^{2024} + (1/x)^{2024}$?
2. Find all positive integers n such that $n = 12S(n)$, where $S(n)$ is the sum of the digits of n.
3. Which is larger 2023^{2024} or 2024^{2023}? How do you know?
4. Suppose that
$$\frac{d}{dx} f(3x) = 6x.$$
What is $f'(x)$?
5. Find all the integers n that can be written as
$$n = \frac{1}{a_1} + \frac{2}{a_2} + \cdots + \frac{20}{a_{20}}$$
for some choice of positive integers
$$a_1 < a_2 < \cdots < a_{20}.$$

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LEVEL I— Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. Let $\mathcal{C}$ be the "unit" circle in the xy-plane with equation $x^2+y^2=1$. Except for $(x,y)=(0,0)$ we say that *the reflection* of the point (x,y) in $\mathcal{C}$ is the point

$$R(x,y) = (x/(x^2+y^2), y/(x^2+y^2)).$$

   (a) Show that $R(R(x,y)) = (x,y)$.

   (b) Describe the curve made up of the reflections of all the points on the line with equation $L(x,y) = y + x - 1 = 0$.

2. Every point on the plane is given one of two colors, red and blue. Show that there exists an isosceles triangle whose two equal sides have length 1 and whose vertices are all of the same color.

3. Concerning fractions:

   (a) Show that for any two positive numbers a and b, we have $\frac{a}{b} + \frac{b}{a} \geq 2$.

   (b) Use the result of part (a) to show that for any three positive numbers a, b, and c, we have $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$.

4. The obtuse triangle $\triangle ABC$ has sides of integer lengths $n, n+1, n+2$.

   (a) What is the value of n? Explain. Hint: there's only one such n.

   (b) What is the cosine of this triangle's largest angle?

5. Peter was 10 the day before yesterday. Next year, he'll be 13. Explain how these statements can both be true.