U.C. MATH BOWL 2024

LEVEL II — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. Find integers $x \leq y \leq z$ so that
$$2^x + 2^y + 2^z = 200$$
Hint: $2^x + 2^y + 2^z = 2^x(1 + 2^{y-z} + 2^{z-x})$.

2. Fill in the squares of the grid with 9 consecutive integers so that the sums of the rows and columns are the numbers indicated in the margin of the table. The integers don't need to be consecutive in the table.

			47
			44
			35
35	42	49	+

3. Every point on the plane is given one of two colors, red and blue. Show that there exists an isosceles triangle whose two equal sides have length 1 and whose vertices are of the same color.

4. Peter was 10 the day before yesterday. Next year, he'll be 13. Explain how this is possible.

5. Show that for any positive integer n
$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}+\sqrt{n+1}} = \sqrt{n+1} - 1.$$

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LEVEL II — Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

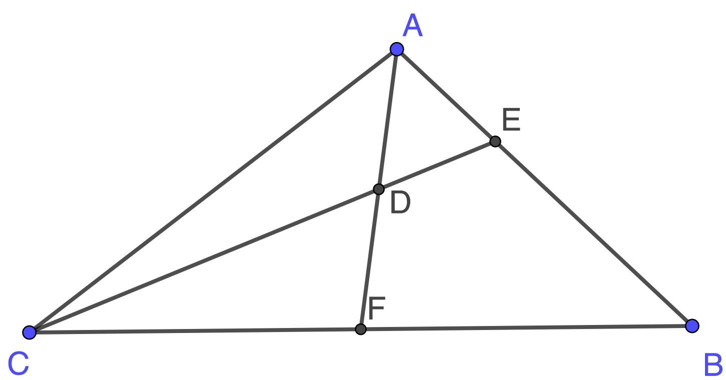
1. Suppose that $A \neq B$ and that

$$A^2 - B = 111$$
$$B^2 - A = 111$$

   What could A and B be?

2. Mike and Katie are playing a game: given a number, the player can do one of the following four operations: multiply this number by 2, divide it by 2, multiply it by 3, or divide it by 3. The goal is to start with the number 12 and arrive at the number 54 in exactly 40 moves. Mike says it's impossible and Katie thinks it can be done. Who is right? Either present a solution or show that one does not exist.

3. Let AF be the median of the triangle ABC, D be the midpoint of AF, and E be the point of intersection of the line CD with the side AB. If $BF = CF = BD$, prove that $AE = DE$. See the figure.



4. Concerning fractions:

   (a) Show that for any two positive numbers a and b, we have $\frac{a}{b} + \frac{b}{a} \geq 2$.

   (b) Use the result of part (a) to show that for any three positive numbers a, b, and c, we have $(a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$.

5. After a teacher graded an exam, two students requested to take a make-up exam on different dates. After the first student took the make-up exam, getting a score of 80, the average score on the exam increased by 1. Three days later, the second student took the make-up exam. His score of 10 decreased the overall class average by 2. How many students took the exam originally?